Quantum Photonics Beyond Conventional Computing

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14. ABSTRACT

This project builds toward a simplified quantum information processing device which, though simpler than a universal quantum computer, uses far fewer resources and can still outperform any classical device for a particular specific task. Using the evolution of photons through linear optical networks for computation, we introduce the Boson Sampling model of linear optical quantum computing, realizing the first universally reconfigurable linear optical circuit, and demonstrate how the device can perform quantum simulations of the evolution of vibrational wave-packets in molecules, suggesting that such an approach could yield the first meaningful quantum simulations which are not possible with classical computation.

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Quantum photonics beyond conventional computing

CONTENTS

I. INTRODUCTION

Quantum computers, exploiting the unique properties of quantum systems, promise to provide exponential speedups for certain hard computational problems with implications across science and technology. A fault-tolerant universal quantum computer capable of outperforming today's classical computers however is thought to require an architecture of tens of thousands of high fidelity physical qubits and is currently a long term experimental goal. For an implementation based on photonics this requires development of arrays of deterministic, indistinguishable single photon sources, fast switching and feed forward of high performance optical circuitry and high efficiency single photon detection. However, it was recently discovered that using far fewer resources, it may be possible to build a much simpler photonic machine which, for a specific task, can still outperform any classical device [1].

This machine harnesses the natural evolution of photons through linear optical networks without requiring switching, feed forward or number-resolved photon counting. Using the integrated quantum photonics platform pioneered at the CQP, the development of many-path passive and reconfigurable circuits combined with integrated photon sources and detectors leaves such a device within reach of short term experimental progress. This report details recent work at the CQP towards this goal and looks ahead to future research. We first introduce the Boson Sampling model of linear optical quantum computing which underpins our approach. Following this we discuss approaches to overcoming the problem of probabilistic photon sources. As we increase the complexity of experiments in terms of number of photons and number of paths, verification will become an important task. We propose and demonstrate new protocols for verifying linear optical devices. Next, we describe the realisation of the first universally reconfigurable linear optical circuit, a versatile device which not only can perform all existing protocols but opens up new avenues of application. Finally, we explore one such avenue by demonstrating how our device can perform quantum simulations of the evolution of vibrational wave-packets in molecules, an application pointing the way towards the first physically meaningful simulations which are beyond the capabilities of classical computing.

II. BOSON SAMPLING

The setup for the boson sampling problem is as follows; n bosons are inputted into the first n modes of a $m \approx n^2$ mode linear network. This network is described by a unitary operator U acting upon the bosonic creation operators of each mode. In Fock notation, a state of bosons in m modes can be written as:

$$|T\rangle = |t_1\rangle_1 \otimes |t_2\rangle_2 \otimes \cdots \otimes |t_m\rangle_m$$

where t_i is the occupation number (i.e. the number of bosons) of the ith mode. The input state is therefore of the form

$$|T_0\rangle = |1\rangle_1 \otimes \cdots \otimes |1\rangle_n \otimes |0\rangle_{n+1} \otimes \cdots \otimes |0\rangle_m$$

The transformation of the network on $|T\rangle$ can be described by a representation of U in the exponentially larger n boson Hilbert space, $\Lambda(U)$. Therefore the state at the output of the system is given by

$$\Lambda(U)|T\rangle = \sum_S \langle S| \Lambda(U) |T\rangle |S\rangle$$

where $|S\rangle$ are the output Fock basis states defined similarly to $|T\rangle$. It can be shown that the probabilities for obtaining an output state $|S\rangle$ are given by

$$p(S|T) = |\langle S| \Lambda(U) |T\rangle|^2 = \frac{|Per(U_{S,T})|^2}{\prod_i t_i! s_i!}$$

Where $U_{S,T}$ is the matrix formed from s_i copies of the ith row of U and t_j copies of the jth column of U. The permanent of a m-by-m matrix with elements $u_{i,j}$ is defined as:

$$Per(U) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} u_{i,\sigma(i)}$$

and can be described as the more common matrix determinant without negative terms.
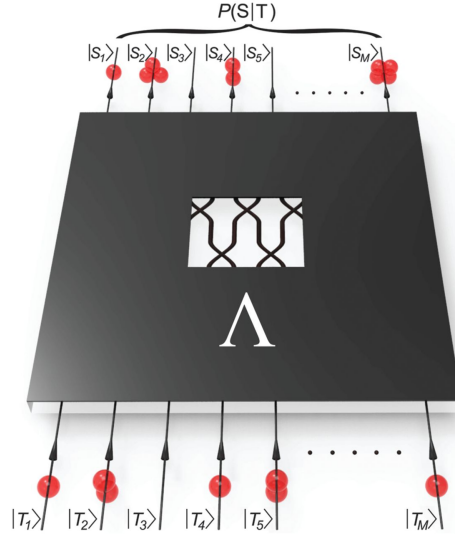
FIG. 1. Model of boson sampling setup, given an input state $|T\rangle$ of n bosons in m modes and transformation Λ, sampling from the distribution $p(S|T)$ is classically hard. Image from [2].

The boson sampling problem can then be defined in the following way: Given a matrix U, let $\mathcal{D}_U$ be the probability distribution over all n-fold output patterns S', where $P[S'] = |\langle S'| \Lambda(U) |T_0\rangle|^2$. Output a sample from the distribution $\mathcal{D}_U$.

The crucial feature is that the transition probabilities are related to the permanents of sub-matrices. For the case of non-interacting fermions, the transition probabilities for states under unitary evolution are governed by matrix determinants, which are efficiently calculable via Gaussian Elimination. However, for non-interacting bosons, the probabilities are governed by matrix permanents, which are known to be classically hard to compute (they are in the computational complexity class $\#P$). This means that exact simulation of a system of non-interacting bosons is very likely to be intractable on any classical device. Of course, in any physical experiment (without error correction) there is no possibility of exact simulation. Instead, experimental error will result in only an approximate simulation of boson sampling. The remarkable result of Ref. [1] is that even with a bounded additive error ϵ, under two likely mathematical conjectures and for the case where the unitary operator of choice is selected uniformly at random over the space of unitaries, approximate simulation is still provably not classically efficient unless the polynomial hierarchy (PH) of complexity classes collapses to its third level, an implication thought very unlikely in computer science.

The boson sampling problem directly maps on to the experimental scenario of inputting n indistinguishable single photons into a linear optical network and measuring n-fold correlated detection patterns at the output. As such, it is thought that a quantum device which implements such a unitary on $O(100)$ optical modes with $O(10)$ photons would outperform any classical device.

A boson sampling experiment consists of three main experimental modules; single photon sources, a linear optical network and single photon detectors. The development of integrated quantum photonics has enabled the development of scalable, phase-stable interferometers which can be used to implement linear optical networks of high complexity and performance [3, 4]. Concurrently, development of superconducting single photon detectors has lead to demonstrations of very high efficiency and low dark count detectors [5]. The main experimental roadblock is currently scaling up the number of single photon sources. One approach to this problem is via multiplexing of current probabilistic photon sources such that with high probability we receive an 'on-demand' photon, in the next section we describe recent work towards this goal.

III. ACTIVE TEMPORAL MULTIPLEXING OF SINGLE PHOTONS

Scalable photonic quantum technologies rely critically on the 'on-demand' release of photons - photons must be made to interfere at precisely the right location and time. While atomic memories may be the ultimate solution to this challenge in the long-term, a promising approach in the short-to-medium term is 'active multiplexing' using optical
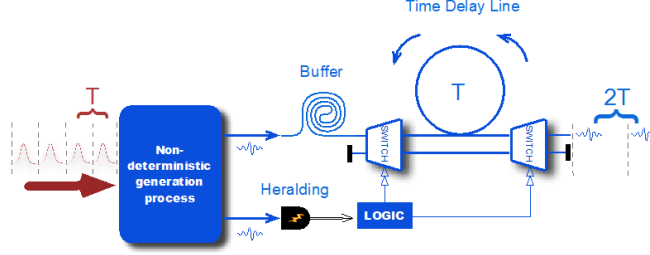
FIG. 2. Principle. A non-deterministic generation process is repeated in time with period T; on heralded success, an active optical switching network and delay lines offset photons into output time bins spaced by an integer multiple of the input period and in sync with the system clock cycle. With a sufficiently low-loss switching network, the generation probability per clock cycle is increased.

switches, delay lines, and feed-forward (Fig. 2). A major benefit of this scheme is that it only uses 'off-the-shelf' optical components that can conceivably be integrated on a single monolithic chip using available fabrication technology.

In Ref [6], we demonstrated active temporal multiplexing and used it to improve the success probability of a heralded single-photon source (HSPS). In a heralded source, a signal and idler photon pair are produced spontaneously; detection of the idler using a single-photon detector will herald the presence of its twin. Using temporal multiplexing, the HSPS is pumped N times per clock cycle with laser pulses spaced by time T. Signal photons are stored in a long delay line buffer as detection signals from the idler arm are analysed. When a single photon is heralded in one of the N time bins, a switching network composed of delay line loops (with lengths of integer multiples of T) is driven into a configuration which offsets the photon into a single spatial-temporal mode (Fig. 2). This technique can be used to boost the single-photon emission probability while keeping the multi-photon contamination low.

Using a bulk and fibre setup (Fig. 3), we demonstrated hybrid temporal and spatial multiplexing of eight photon bins to enhance the heralded photon emission probability by up to 79% compared to the non-multiplexed sources. Although our demonstration was limited by the loss and speed of the switches, our proof-of-principle points the way to future implementations in an integrated platform that will be invaluable for scaling quantum technology.
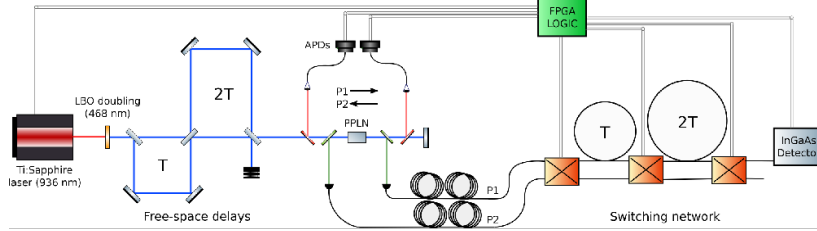

FIG. 3. Experimental setup. Pulses from a femtosecond laser are up-converted using an LBO crystal, split into four copies using free-space delay lines, and passed twice through a PPLN crystal for down-conversion. Following separation of the photon pairs and pump using filtering, the heralding signals are analysed by an oversampling FPGA while the signal photons are stored in long fibre delays. The FPGA configures the switching network to deliver the generated signal photons into a single spatial and temporal mode. P1 and P2 indicate Pass 1 and Pass 2.

Active multiplexing is a potential hardware solution to the photon source problem, the next section describes theoretical developments that have lead to a new modification of the boson sampling protocol which allows for a much better scaling of photon sources with current technology, pointing the way towards larger-scale demonstrations.

IV.   BOSON SAMPLING FROM A GAUSSIAN STATE

The most common single photon sources used for quantum information processing work via spontaneous pair generation, where a pump field is incident upon a nonlinear material producing a two-mode squeezed vacuum state. One mode is then sent directly to a detector which heralds the existence of a photon in the other. However, since these sources are spontaneous, the probability for producing n photons with n sources drops exponentially with
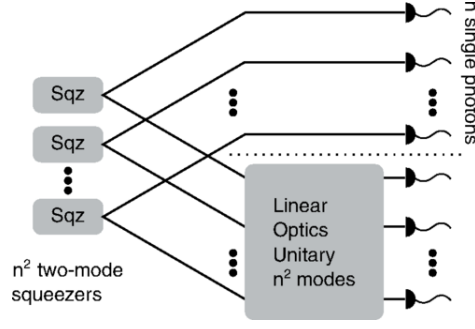
5

FIG. 4. Boson sampling from a gaussian state. n^2 two-mode squeezers are used as single photon sources, with half the modes sent to heralding detectors and the other half sent through a linear optical unitary.

n. While deterministic single photon sources based on two-level emitters or multiplexing as described above are in development, they are not currently a short term solution. However, a simple alleviation to this problem, which retains the complexity implications of the original protocol is to add many more sources so that there is a spontaneous source at each of the n^2 inputs to our circuit, rather than just the first n [7].

The state of a two-mode squeezer can be written as

$$\sqrt{1-\chi^2}\sum_{p=0}^{\infty}\chi^p\,|p\rangle_1\,|p\rangle_2$$

where $0 \leq \chi < 1$ is the squeezing parameter. If we have n^2 such sources, as shown in Fig. 4, this leads to a probability of getting the usual $|T_0\rangle$ input state of

$$P(T_0) = \chi^{2n}(1-\chi^2)^{n^2}$$

The reason it is hard to even approximately sample a boson sampling distribution is because the unitary U is random, and therefore its sub-matrices in the first n columns are approximately gaussian. The proof of [1] relies on the conjecture that estimating the permanents of these gaussian matrices is computationally hard. Since U is random however, any submatrix will look approximately gaussian and so any input state of n photons in different modes should be hard to sample from. Therefore we can adapt the usual boson sampling problem to include sampling over both outputs and inputs and retain the results for classical hardness. Now our n^2 sources help us, since we don't care what the input state is as long as we get n photons. This occurs with a probability

$$P(T) = \binom{n^2}{n}\chi^{2n}(1-\chi^2)^{n^2}$$

which, for the optimal squeezing parameter $\chi = \chi_{\max}$ leads to an asymptotic behaviour of the form

$$P(n|\chi_{\max}) \propto \frac{1}{\sqrt{n}}$$

A vast improvement on the exponentially poor scaling of only n sources. Recently, the first demonstration of this approach has been experimentally implemented [8].

V.   VERIFICATION OF BOSON SAMPLING

As experimental demonstrations of boson sampling increase in scale and complexity a natural question to ask is how to verify that a purported boson sampler is truly performing non-classical computation. In contrast to quantum algorithms such as Shor's factoring algorithm, where the solution can be efficiently classically verified, it is not currently known how to check whether a large-scale boson sampler is operating correctly. In the absence of a mathematically rigorous approach to verification, the task of assessing correct operation then becomes one of collecting sufficient experimental evidence in different regimes.
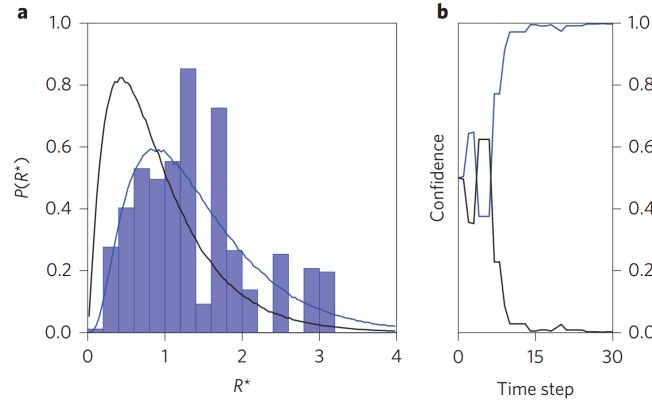
6

FIG. 5. Verification of boson sampling against the uniform distribution. a) the expected PDF for $R^\star$ for sub-matrices chosen from the boson sampling distribution (blue line) and the uniform distribution (black line). The bars show a histogram of the experimental data. b) Dynamic updating of confidence in sampling form the boson sampling distribution rather than the uniform distribution.

One claim, detailed in Ref [9], is that the output distribution of a boson sampler is operationally indistinguishable from simply drawing samples at random from a uniform distribution. However, as was shown in Ref [10], it is possible to use an efficiently calculable discriminator to verify that the statistics are not drawn from a uniform distribution. This discriminator is given by the product of squared row 2-norms of an $n \times n$ sub-matrix M calculated as $R^\star = \prod_i^n R_i$ where for each row $R_i = \sum_{j=1}^n |a_{i,j}|^2$. Using an integrated silicon nitride photonic chip with a nine-mode array of directional couplers and fixed phase shifts implementing a random unitary, we calculated $R^\star$ for 434 events using $n = 3$ photons, shown in Fig. 5 [11]. We used bayesian model comparison to update our relative confidence that the samples were drawn from either the boson sampling distribution or the uniform distribution, finding a final probability that the samples were not drawn from the uniform distribution of $1 - 10^{-35}$.
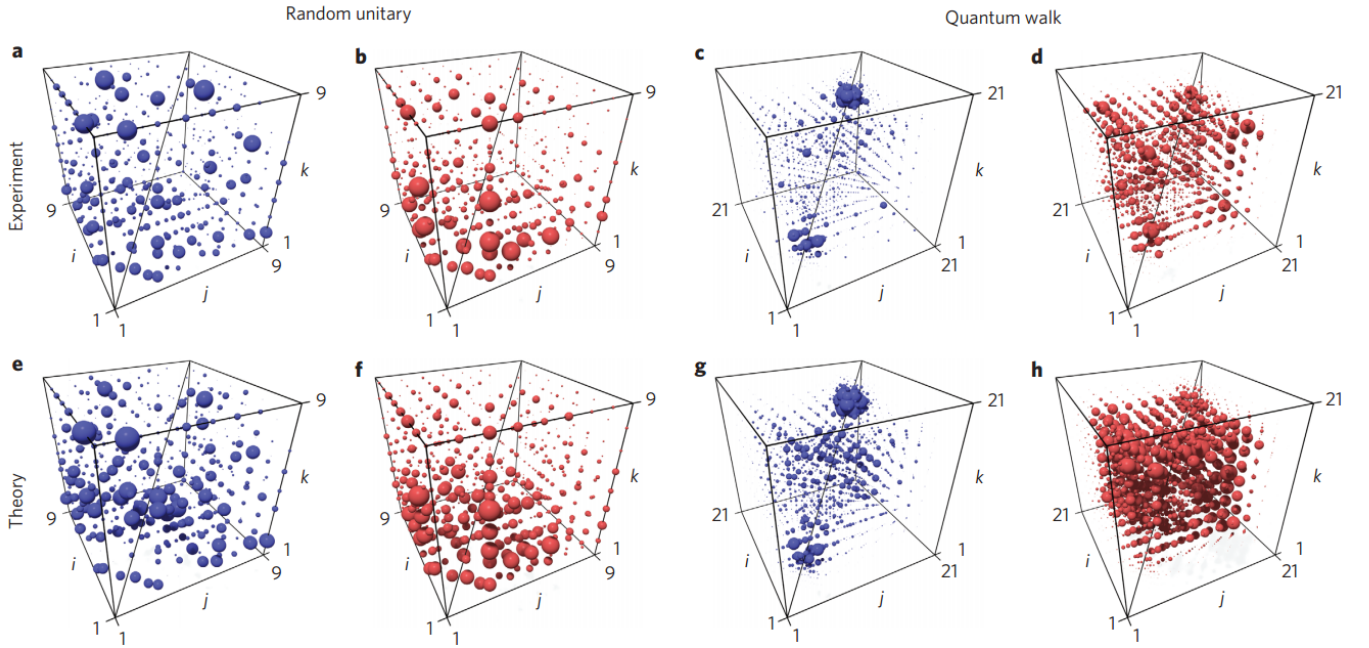
FIG. 6. Bosonic correlations for indistinguishable and distinguishable photons. Radii of spheres at coordinates (i, j, k) are proportional to finding photons in modes i, j and k. a,b) indistinguishable (blue) and distinguishable (red) experimental correlation data for the random unitary chip. c,d) same for the quantum walk chip, where bosonic clouds form in the indistinguishable case. e,f) theoretical distributions for the random unitary and g,h) for the quantum walk.

One of the main experimental challenges in boson sampling is producing photons which are indistinguishable in all non-spatial degrees of freedom (e.g. polarisation, frequency). If the photons are distinguishable from each other, no quantum interference takes place, leading to a classically tractable distribution. Therefore this is a more physically motivated distribution which needs to be ruled out. The method introduced in [11], approaches this problem by considering, instead of a random unitary, a highly structured unitary for which certain parts of the n photon probability distribution can be determined efficiently. After experimentally confirming correct operation in this configuration, we then imagine the circuit can be continuously tuned to a random unitary whilst maintaining this operation.

For a proof-of-principle demonstration of this approach, we used two circuits, the 9x9 random unitary described above and a quantum walk chip comprising of an array of 21 evanescently coupled waveguides fabricated in silicon oxynitride, and physically swapped them. The unitary description of the quantum walk device is given by exponentiating a nearest-neighbour Hamiltonian resulting in a well structured matrix. The reason this unitary is chosen is that it exhibits a behaviour we term 'bosonic clouding' whereby indistinguishable photons tend to cluster in nearby modes in a superposition around two locations, a tendency not seen in the classical case.

Fig. 6 shows experimental correlations between three photon detections for the random unitary, where there is little structure in either case and the quantum walk where bosonic clouding can clearly be seen in the indistinguishable case but not in the distinguishable case. To quantify the amount of clouding in a given experiment, we construct a clouding metric $\mathcal{C}$. For a given trial i, if t and b are the numbers of photons found in the top and bottom halves of the device we calculate $c_i = 2|(t-b)/(t+b)| - 1$, the clouding metric is then calculated as the average of this value over all n trials, $\mathcal{C} = \sum_i c_i/n$. For 3 photons we found a value of $\Delta\mathcal{C} = \mathcal{C}_Q - \mathcal{C}_C = 0.138 \pm 0.014$ revealing the presence of clouding. For states of 5 photons, where the Hilbert space of the system has a dimension of over 50,000, we still find such a separation $\Delta\mathcal{C} = 0.137 \pm 0.041$.

VI. QUANTUM-ENHANCED TOMOGRAPHY OF UNITARY PROCESSES

Another important form of verification and a fundamental task in photonics is to characterise an unknown optical process, defined by properties such as birefringence, spectral response, thickness and flatness. Among many ways to achieve this, single-photon probes can be used in a method called quantum process tomography (QPT). However, the precision of QPT is limited by unavoidable shot noise when implemented using single-photon probes or laser light. In situations where measurement resources are limited, for example, where the process (sample) to be probed is very delicate such that the exposure to light has a detrimental effect on the sample, it becomes essential to overcome this precision limit. In Ref [12] we devise a scheme for process tomography with a quantum-enhanced precision by drawing upon techniques from quantum metrology. We implement a proof-of-principle experiment to demonstrate this scheme - four-photon quantum states are used to probe an unknown arbitrary unitary process realised with an arbitrary
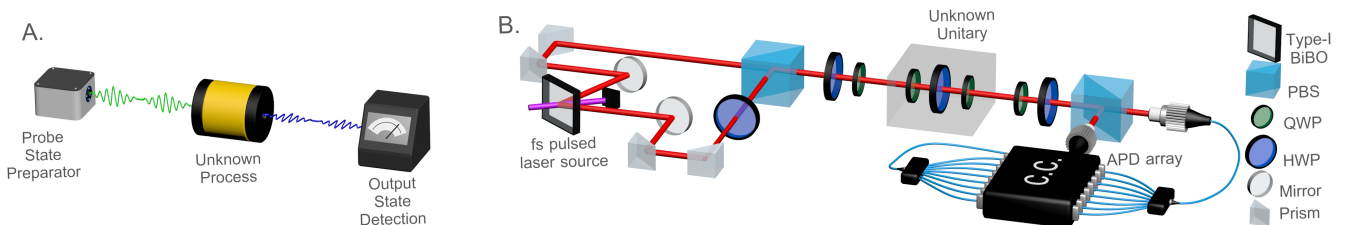

FIG. 7. Experimental setup. (a) Standard procedure for quantum process tomography. This procedure can be considered in three stages: preparation of a series of probe states, interaction of the unknown process with the probe, and detection of the output in a selection of measurement bases. Based on the measurement outcomes, a mathematical map corresponding to the unknown process can be reconstructed. (b) Experimental setup for probing the unknown unitary processes. An 80 MHz pulsed Ti:sapphire laser centred at 808 nm is up-converted to 404 nm and then focused onto a bismuth borate (BiBO) crystal, phase-matched for Type-I SPDC, creating non-collinear degenerate horizontally polarised photon pairs at 808 nm. After converting one arm to vertical polarisation using a half- wave plate (HWP), the two arms are combined by a polarising beam splitter (PBS). The resulting state is passed through the unknown unitary and then measured in the H/V basis. Approximate photon-number counting (see manuscript for details) is implemented on each polarisation mode using a 1-8 fan-out array onto avalanche photodiode detectors (APDs). By using different wave plate settings before and after the unknown unitary, the basis for the probe state and measurement can be changed to D/A and R/L. The setup is pre-calibrated using laser light.
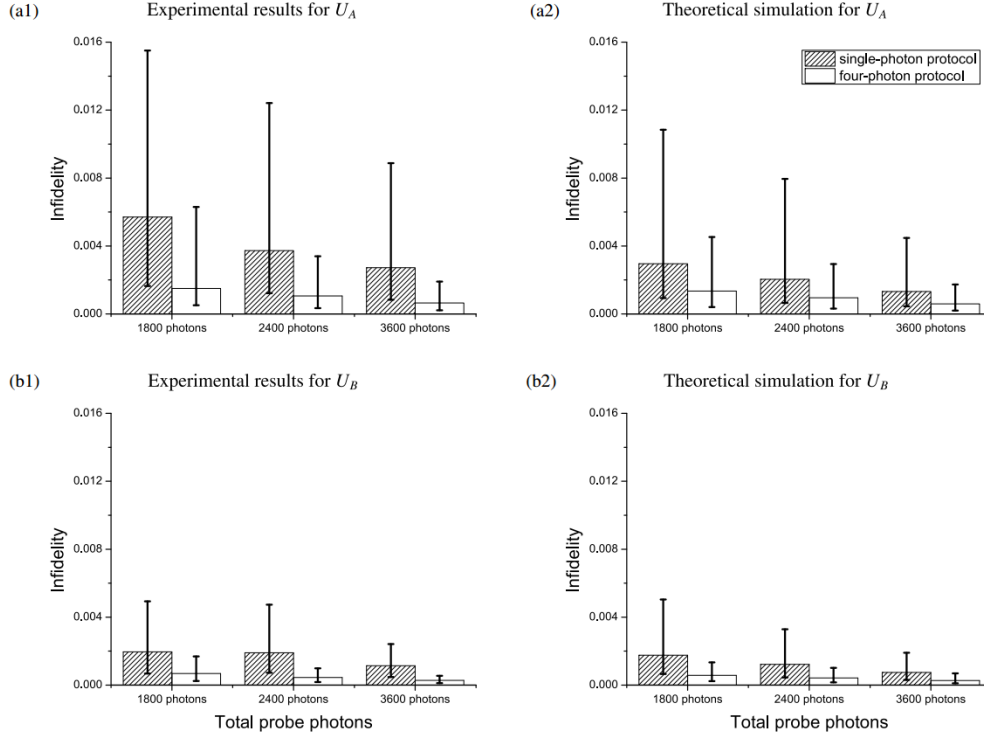
8

FIG. 8. Experimental performance of the quantum-enhanced tomography protocol using four-photon states. In this figure, the standard QPT protocol (diagonal fill) using single photons and our method using four photons (unpatterned) are compared, for two example unitaries. To make a fair comparison between the two protocols, the same total number of probe photons are assumed (1800, 2400, and 3600 for all measurements together). For each case, the mean infidelity is illustrated, for a series of experimentally derived estimates, where the infidelity is defined relative to central estimates of the reconstructed examples, derived from accumulated experimental data. Asymmetric lower- and upper-half error bars are used, reflecting the distribution of infidelity values. The data show that the four-photon version of the protocol achieves greater accuracy and precision than standard QPT with single photons.

polarisation rotation. Our results show a substantial reduction of statistical fluctuations compared to traditional QPT methods - in the ideal case, one four-photon probe state yields the same amount of statistical information as twelve single probe photons.

In order to perform a convincing demonstration of boson sampling, an experiment should be able to implement any arbitrary unitary U given to it as an input. Furthermore, this device would be able to implement the verification procedure described in the previous section as well as opening up a range of new applications. The next section describes the first experimental demonstration of such a device.

VII.   UNIVERSAL LINEAR OPTICS

Any linear optical network of beamsplitters and phase-shifters is described by a unitary operator. It has been shown that any unitary can be implemented with a specific array of $\approx m^2/2$ two-mode primitives with two parameters each [13] (see Fig. 9a). In integrated photonics, beamsplitters are realised by directional couplers, where two waveguides are brought close together so that their evanescent fields overlap, and phase shifters can be implemented by tuning the local refractive index in one waveguide relative to the others. In Ref. [14] we demonstrate the longstanding goal of a single reconfigurable circuit (LPU) which can be programmed to implement any unitary using this decomposition.

The experimental setup, shown in Fig. 9b, consists of a single photon source, which uses spontaneous parametric downconversion to generate up to six photons, a photon detection system of 12 single photon avalanche diodes and an integrated silica-on-silicon waveguide circuit across six modes with 15 Mach-Zehnder interferometers and 30 thermo-optic phase shifters. The phase shifters are resistively heated by the application of voltage, causing the refractive index to change accordingly. By tuning the voltage across all the heaters, any possible combination of phases can be
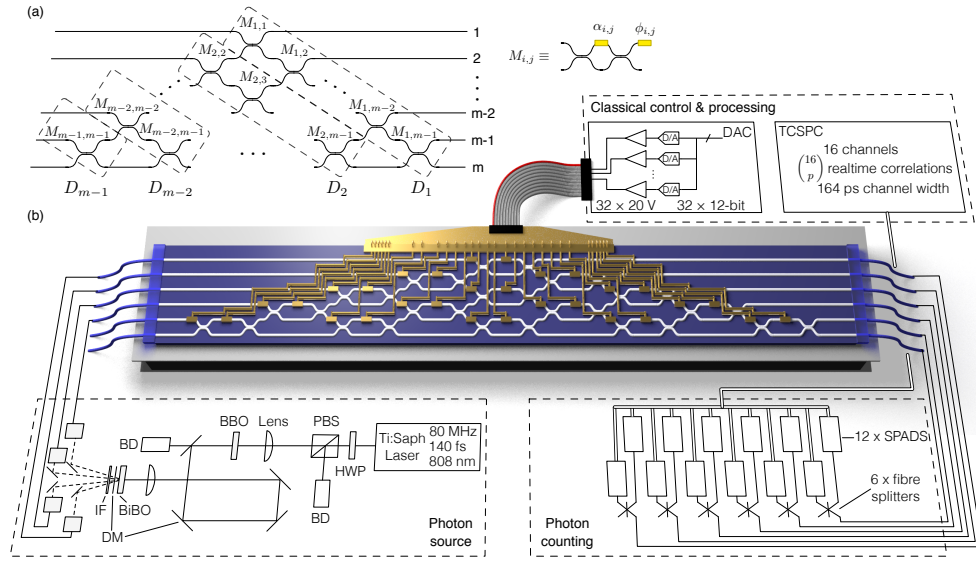
9

FIG. 9. Universal linear optical processor (LPU). a) decomposition of a unitary for an m-mode circuit to realise any linear optical operation. b) multi-photon ensembles are prepared via a source using spontaneous parametric down conversion. The silica-on-silicon chip consists of 15 Mach-Zehnder interferometers controlled by 30 thermo-optic phase shifters set by a digital-to-analogue converter (DAC). Photons are detected using up to 12 single photon avalanche diodes (SPADs) and counted using a time-correlated single-photon counting module (TCSPC).

implemented corresponding to any possible unitary.

We configured the device to implement 100 different instances of boson sampling for 1,2 and 3 photons shown in Fig. 10. To implement the random unitaries, we used the scheme of Ref [15] to directly configure our device by choosing each of the experimental parameters from a set of probability density functions. The averages over all
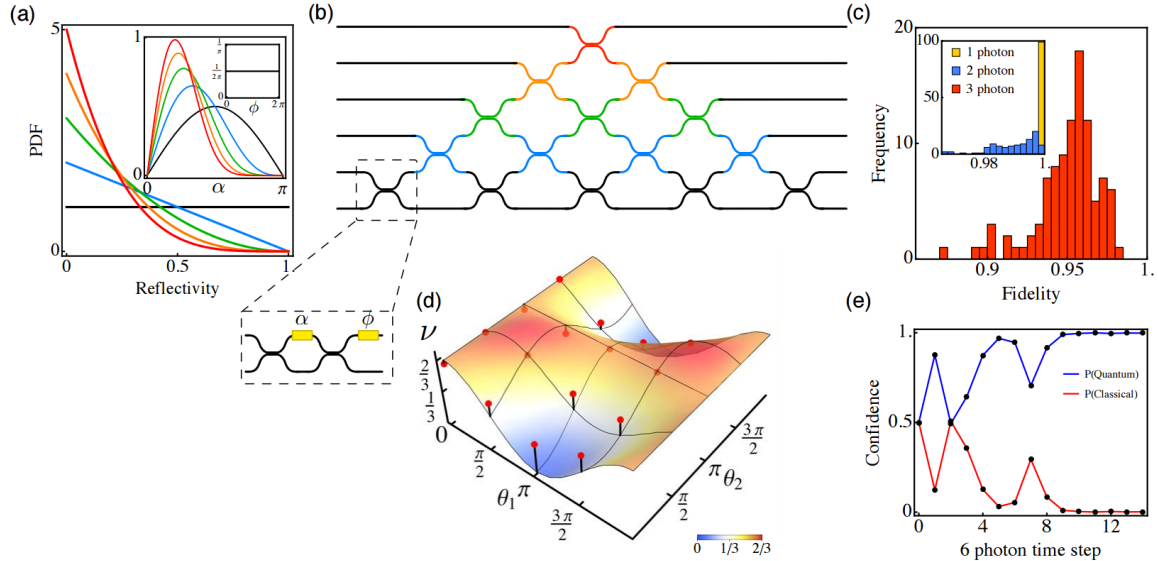
FIG. 10. Boson sampling over a range of unitaries. a) Random unitaries can be implemented by drawing the phases from the probability density functions(pdfs) corresponding to the elements in b). c) A histogram of statistical fidelities for 100 boson sampling experiments for three photons, two and one photon histograms inset. d) experimental points showing violations ν of the ZTL over $F_6^{(2)}(\theta_1, \theta_2)$ matrices. e) dynamic updating of confidence in sampling from indistinguishable(quantum) and distinguishable(classical) distributions for a six photon input.

10

hundred unitaries of the statistical fidelity of the resulting probability distributions, defined as $\mathcal{F}_s = \sum_i \sqrt{p_i q_i}$, were found to be $0.999 \pm 0.001, 0.990 \pm 0.007$ and 0.950 ± 0.020 respectively.

The ability to arbitrarily tune the device also allows for a full demonstration of the verification procedure outlined in the previous section. For this demonstration we used the discrete Fourier transform (FT) matrix as the structured unitary. The zero transmission law (ZTL) [16] uses the symmetry of the FT matrix to determine that the probabilities for most of the exponentially growing number of correlated photon detections are strictly suppressed for this case. Since this suppression is caused by multi-photon quantum interference, it has been proposed as a stringent assessment of boson sampling devices [17]. We implemented a range of examples from a two-parameter family $F_6^{(2)}(\theta_1, \theta_2)$ of complex Hadamard matrices (unitaries for which the absolute squared value of every element is $1/N$), where the FT matrix corresponds to $F_6^{(2)}(\pi, 0)$. Using 3 photons, we calculated the experimental violation of the ZTL as $\nu = N_s/N$, the ratio of suppressed events N_s to the total number of events N, the results of which are seen in Fig. 10d.

Finally, in Fig. 10e we demonstrated that the chip maintains six photon indistinguishability by inputting 3 photons into each of the top two waveguides of the FT matrix and using bayesian model comparison to compare our relative confidence of sampling from the correct distribution, finding a final confidence of $> 99\%$ that we are sampling from the correct distribution .

The ability to implement any circuit opens up a range of new applications and experiments. In the final section we describe ongoing work using the framework of boson sampling to realise a new form of quantum simulator which could lead to the first quantum simulations that exceed the capabilities of classical devices.

VIII.   QUANTUM SIMULATION OF HARMONIC PHONONS

Simulating the evolution of quantum systems is in general an intractably hard computational task making it a key application of quantum technology for chemistry, biology and material science. In contrast to a digital simulation on a universal quantum computer, analogue quantum simulators use the specific properties of quantum systems in the lab to simulate similar systems in nature which are physically less accessible or controllable. By exploiting the natural Hamiltonian of the given system the resources required to perform a non-trivial quantum simulation can be greatly reduced.

A common approach to simulating the dynamics of interacting particles involves finding a transformation which maps the problem to one of non-interacting particles. One such example is the case of phonons, where the harmonic approximation (i.e. assuming small displacements) of an atomic Hamiltonian gives rise to collective vibrational quasi-particles which act as non-interacting bosons with well defined energies. The ability to implement and reconfigure

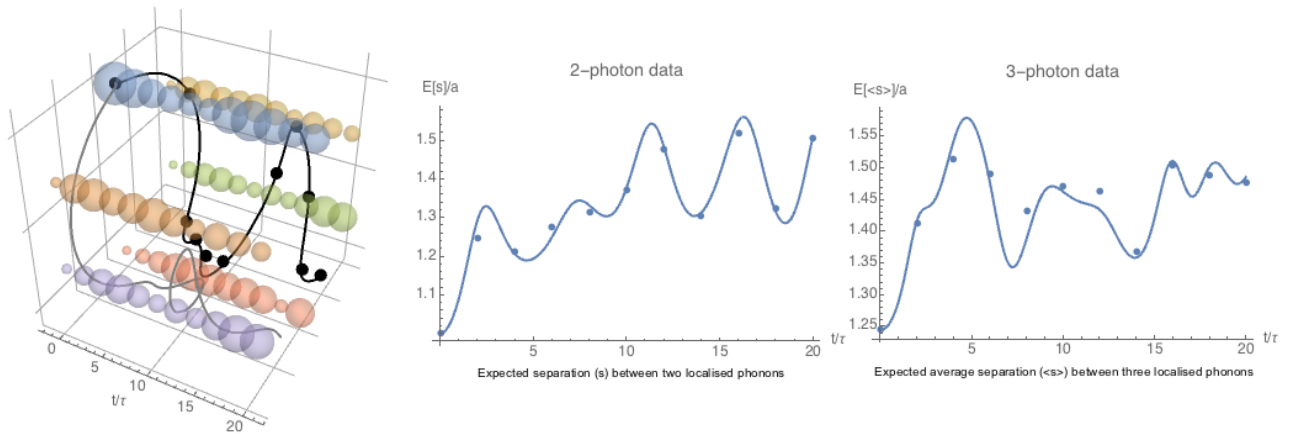FIG. 11. Simulation of localised phonons in a ring of atoms. a) relative volume of bubbles indicates the relative probability of detecting a phonon localised in each of the atomic sites of the ring. Black line indicates the expectation value of the angular position of a localised phonon (grey if we define the anti-clockwise angular position). b) expected separation between two localised phonons. c) same for three phonons.

any m dimensional unitary transformation on n photons, as described in the previous sections, implies the ability to simulate any m mode non-interacting Hamiltonian on a system of n bosons [18]. Simulation is then accomplished by tuning the optical elements to implement the unitary operation $U(t) = e^{-iHt}$ that corresponds to the Hamiltonian H evolved to the desired time t. The evolution of the system can then be observed by reconfiguring the circuit to realise a sequence of unitaries for each of the desired time steps and tracking the evolving probability distribution over the modes. Since these distributions are governed by permanents, there is reason to believe that simulating this evolution for many physical scenarios is classically inefficient.

The textbook model of a lattice of atoms describes how phonons arise. Consider m particles with mass M at position x and connected by springs of constant C and length l in a ring. The transverse displacement of the particle at x is q_x and its momentum is p_x, leading to a Hamiltonian

$$H = \frac{1}{2}\sum_{x=1}^{N} p_x^2/M + C(q_{x+1} - q_x)^2$$

This system can be simplified by converting between p_x and q_x and their respective Fourier transform coordinates P_k and Q_k. In this basis, the motion of the ring is decomposed into the normal modes, or in the quantum case phonons, of the system with corresponding frequencies $\omega_k = \sqrt{2C/M}\sqrt{1 - \cos(kl)}$. When applying this model to quantum systems, second quantisation introduces creation and annihilation operators $a_k^\dagger$ and a_k defined on these phonon modes and produces the Hamiltonian $H = \sum_k (a_k^\dagger a_k + \frac{1}{2})\hbar\omega_k$ which describes a quantum field theory of non-interacting bosons. This Hamiltonian is diagonal and as such its evolution is trivial, however if we consider calculating correlations from an arbitrary measurement basis $M = \sum_i |i\rangle\langle m_i|$ for many phonons prepared in another basis $P = \sum_i |p_i\rangle\langle i|$ then we can simulate the evolution $U'(t) = M.e^{iHt}.P$. For the case of the ring, if we choose M and P to be the FT matrix, this corresponds to initially localising phonons at all the sites where photons are inputted and then simulating their spatial distribution over time as depicted in Fig. 11.

Using this framework, in Fig. 12 we address larger scale systems containing six vibrational modes. The number of vibrational modes of a molecule with n atoms is generally $3n-6$. We simulated the case of H_2CS, a four atom molecule with six vibrational modes, in two bases; the localised basis of phonons around certain bonds of the molecule, and random bases at preparation and measurement in order to put the simulation into the regime of boson sampling. Our approach can also be used for larger molecules to simulate time evolution of correlations in a subspace of vibrational modes. We show this for the case of NMA in which the superpositions of a six mode subset of its 33 vibrational modes give rise to localised backbone vibrations.
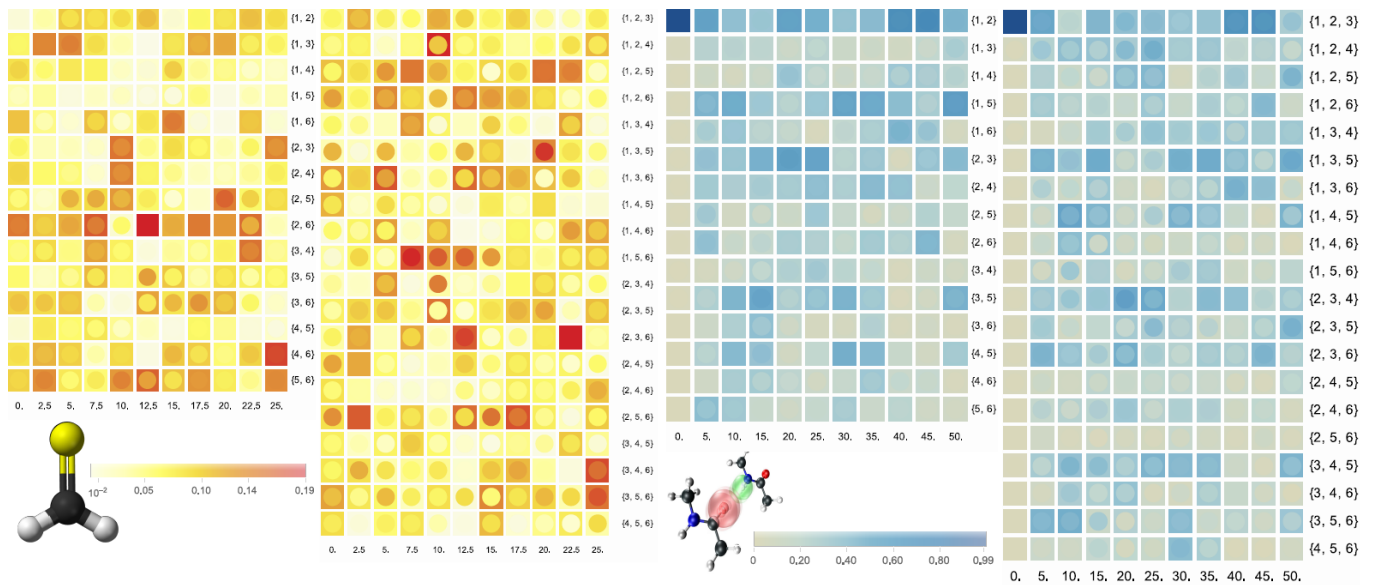

FIG. 12. Simulation of phonons in H_2CS and NMA. a) Time evolution of the coincidental detection of two and three phonons in H_2CS with random preparation and measurement bases. Coloured squares are theoretically expected values, overlaid disks represent experimental data. b) Same for NMA with localised backbone vibrations.

12

We have shown that photons propagating in a fully reconfigurable linear optical circuit can function as a simulator which is universal for the class of non-interacting bosons. Due to the relatively low experimental resources required, and the theoretical evidence that such systems are not efficiently simulatable, this suggests that such an approach could yield the very first meaningful quantum simulations which are not possible with classical computation.

IX. CONCLUSIONS

Here we have described the rapid progress achieved at the CQP towards the major goal of an integrated photonic system capable of outperforming classical computers. We have demonstrated experimental and theoretical solutions to improve the scalability of single photon sources and proposed new verification protocols clearing the way for large-scale multi-photon experiments. In addition, through the development of universally reconfigurable circuits we have opened up the possibility of performing physically meaningful, classically intractable simulations with minimal quantum resources.

Combining the development of integrated on-chip photon sources [19] with the source-boosting schemes described in Sections II and III will enable the monolithic implementation of arrays of single photon sources in order to produce the 10s of photons needed for photonic simulations beyond the capability of conventional computers. Integrating single photon detectors on-chip is also an immediate goal as this will allow for high-efficiency detection and dramatically reduce the detrimental effects of photon loss. Alongside these developments we will look to increase the size of reconfigurable circuitry, allowing for larger-scale and more sophisticated experiments exploring large dimensional Hilbert spaces where verification protocols will be essential.

Work must now be done to further develop theory for physical scenarios where photonics simulators will find application. A natural extension to the molecular simulations described in Section VIII is to introduce some non-linearities into the photon sources (e.g. by squeezing) or circuit (e.g. by heralded quantum logic), which can be mapped to vibrational couplings in molecules. Such a system would allow for an improvement to the accuracy of the molecular model, opening up new and important possibilities across science, such as new drug design.

X. LIST OF PUBLICATIONS SUPPORTED BY USAF

- J. Carolan *et al.*, 'On the experimental verification of quantum complexity in linear optics', *Nat. Photon.*, **8** , 621-626, (2014)

- A. Lund *et al.*,'Boson Sampling from a Gaussian State', *Phys. Rev. Lett.*, **113**, 100502, (2014)

- X-Q Zhou *et al.*, 'Quantum-enhanced tomography of unitary processes', *Optica*, **2**, 510-516, (2015)

- J. Carolan *et al.*,'Universal linear optics', *Science*, **349**, 711-716, (2015)

- N. Russell *et al.*, 'Direct dialling of Haar random unitary matrices' *arXiv:1506.06220*, (2015)

- G. Mendoza *et al.*, 'Active Temporal Multiplexing of Photons', *arXiv:1503.01215*, (2015)

- E. Martin-Lopez *et al.*, 'Quantum Simulation of Harmonic Phonons', *In Preparation*

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